ON THE MEASUREMENT OF SPHERICAL ABERRATION CONSTANTS OF THE PROJECTOR LENS OF AN ELECTRON MICROSCOPE

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ABSTRACT Magnification of the projector lens of Siemens Elmiskop I was calibrated at its different excitation energy with the help of a grating replica of known spacing. From the measured radii of the images of the aperture fitted to the projector lens, the spherical aberration constants of the lens at its different magnifications were determined, and compared with similar values of other workers.

INTRODUCTION

It is well known that in the electron microscope, the spherical aberration of the projector lens gives rise to distortion of the final image, specially at low magnification (Zworykin *et al*, 1948). In the present paper, a method is described by means of which the spherical aberration constants of a typical projector lens has been evaluated from a measurement of the distortion present in the final image.

THEORETICAL CONSIDERATIONS

Let us consider a simplified ray diagram shown in Fig. 1, where L represents the projector lens, MN the intermediate image formed in front of the projector lens by the objective lens, and SC is the final screen or photographic plate. If

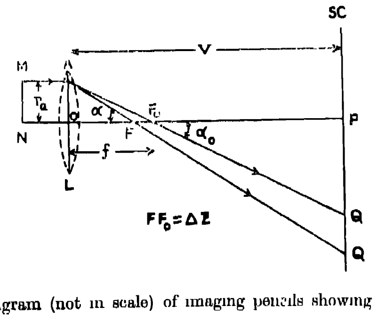

Fig. 1. Schematic diagram (not in scale) of imaging pencils showing the effect of spherical aberration

the projector lens were free from spherical aberration, all axial as well as marginal rays from MN which are parallel to the optic axis, would pass through the focal point of the lens F_0. Hence the object point M would have Q as its image point on the screen But due to spherical aberration of the lens L, the marginal ray MA would cross the axis at F which is nearer to the lens than F_0 Thus the point Q' will be really the image of M and QQ' will be the amount of distortion of the point Q. From Fig. 1

$$QQ' = PQ' - PQ = PF \tan\alpha - PF_0 \tan\alpha_0$$

$$= (PF_0 + \Delta Z)\cdot OA/OF - PF_0 \cdot OA/OF_0$$

$$= (V - f + \Delta Z)\cdot \frac{r_a}{f - \Delta Z} - (V - f)\frac{r_a}{f}$$

$$= r_a \left[\frac{V - f + \Delta Z}{f}\right]\left(1 + \frac{\Delta Z}{f}\right) - (V - f)\cdot\frac{r_a}{f}\ \text{, as } \Delta Z < f$$

$$= \frac{r_a \cdot \Delta Z \cdot V}{f^2} = \frac{r_a \cdot \Delta Z}{f}\cdot\frac{V}{f} = \frac{r_a\,\Delta Z}{f}\,(M_p - 1)$$

$$= \frac{r_a \cdot \Delta Z}{f}\cdot M_p, \quad \text{if} \quad M_p >> 1 \qquad \dots (1)$$

where r_a is the radius of the aperture fitted to the projector pole piece, V is the distance of the screen from the pole piece centre, M_p is the magnification of the projector lens Here ΔZ is the amount of longitudinal spherical aberration, which can be expressed, according to Liebmann (1949), as

$$\Delta Z = C_s \cdot \left(\frac{r_a}{f}\right)^2 \qquad \dots (2)$$

where C_s is the spherical aberration constant of the lens. From the equations (1) and (2), the amount of distortion is given by

$$QQ' = d = C_s \cdot M_p \left(\frac{r_a}{f}\right)^3 \qquad \dots (3)$$

Now, the focal length of a projector lens is given by

$$f = V/(M_p + 1) = \frac{V}{M_p}, \quad \text{if } M_p >> 1 \qquad \dots (4)$$

Therefore equation (3) can be written as

$$C_s = \frac{d}{(M_p)^3} \cdot \left(\frac{V}{r_a}\right)^3 \qquad \dots \quad (5)$$

The method underlying the measurement of d is as follows Let the illuminated circle in Fig. 2 represent the image of the aperture fitted with the projector pole

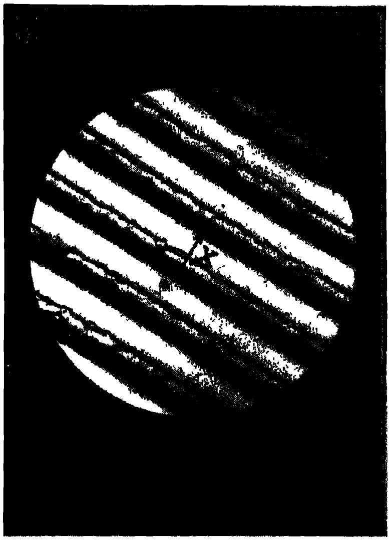

Fig. 2 A typical micrograph of a grating replica

piece. The lines thus projected within the illuminated circle are the rulings of a grating replica whose mean spacing width W is known If X, the width of the central spacing in the micrograph, be measured, then

$$M_T = X/W \qquad \dots \quad (6)$$

and

$$M_p = M_T/M_0 \qquad \dots \quad (7)$$

where M_T, M_p and M_0 are the total, the projector and the objective lens magnifications respectively. Hence the aberration-free radius of the projector aperture in the micrograph should be given by

$$R_0 = r_a \cdot M_p \qquad \dots \quad (8)$$

If the radius of the aperture as measured on the micrograph be R_a, then the amount of distortion d, at the projector lens magnification M_p and for the aperture radius r_a, is obviously expressed as

$$d = R_a - R_0 \qquad \ldots \quad (9)$$

Hence, working with an aperture of known bore radius r_a, and knowing the distance of the screen from the pole piece centre of the projector lens, the projector lens magnification M_p and the corresponding distortion d, the spherical aberration constant C_s of the lens can be found out with the help of equation (5).

EXPERIMENTAL METHOD AND RESULTS

A grating replica of known grating constant was chosen as the object and the microscope was operated at 60 kv at a constant objective lens magnification of 75 15X To test the uniformity of the grating spaces, several micrographs were taken with different grating space at the centre of the field of view at a constant magnification of both the objective and projector lenses From about 30 micrographs thus taken, the widths of the grating spacings were found to vary within ± 4 per cent of the mean value. Then a particular grating space was brought to the centre of the field of view and micrographs were taken at several settings of the projector lens current, which were noted and are shown in column 1 in Table I. Since the object occupies a very small zone around the optic axis, paraxial imaging condition may be assumed and the images of the grating space in the centre of the field of view may be considered to be free from aberration The width of the central spacings of the micrographs at the various magnifications were measured. Dividing these widths by the grating constant W, the total magnification M_T, of the different micrographs were estimated, and by further dividing these total magnifications by the objective lens magnification M_0, the corresponding magnifications due to the projector lens M_p were calculated as shown in column 2 of Table I

The radii R_a of the circular contour of the micrographs, taken at different projector lens magnifications, were measured and tabulated in column 3 of Table I With the help of equation (8), the aberration-free radii R_0 of the micrographs were calculated and entered in the 4th column of Table I, where, in column 5, the respective amount of distortions as obtained from equation (9) are also shown With these values of M_p and d of the columns 2 and 5 respectively in Table 1 and with the fixed value of $V = 367$ mm and $r_a = 0.1925$ mm the corresponding C_s values were calculated and tabulated in column 6 of Table I

DISCUSSION

In the literature, C_s is usually expressed in terms of the pole piece bore radius R as a function of the lens excitation NI For the lens excitation, however, a parameter k^2 is used where k^2 is given by

TABLE I

$r_a = 0.1925$ mm, $M_0 = 75.15$, $V = 367$ mm

Projector lens current I in mA	Magnification of the projector lens $M_p = M_T/M_0$	Measured radius R_a in mm	Aberration free radius $R_0 = r_a \times M_p$ in mm	Distortion $d = R_a - R_0$ in mm	Spherical aberration constant $C_s = \frac{d}{(M_p)^4} \cdot \left(\frac{V}{r_a}\right)^3$
1	2	3	4	5	6
75	76.56	16.0	14.74	1.26	244.5
90	104.8	22.1	20.20	1.90	106.4
100	128.3	26.9	24.71	2.19	54.80
105	140.9	29.6	27.14	2.46	41.48
118	170.0	35.6	32.73	2.87	23.81
125	191.8	40.2	36.92	3.28	16.55
130	196.4	41.2	37.81	3.39	15.53
135	207.5	43.5	39.94	3.56	13.09
140	226.0	47.1	43.52	3.88	10.17

$$k^2 = \beta (NI)^2/\phi_r \qquad \text{.. (10)}$$

where NI is the ampere turns of the lens winding, $\phi_r = \phi(1 + 0.978 \times 10^{-6}\phi)$ is the relativistically corrected accelerating voltage, and β is a factor which depends on pole piece parameter viz. bore radius R or bore diameter D, and spacing S. Magnitudes of β for various values of S/D have been given by Liebmann *et al.* (1951), from which for our case, with $D = 1.0$ mm, $S = 1.0$ mm, the corresponding value of β is 0.006, and since the beam energy is 60 kv, equation (10) can be written as

$$k^2 = 9.448 \times 10^{-8}(NI)^2 \qquad (11)$$

The number of coils in our case is 12,650, hence from equation (11) k^2 values were calculated with different values of lens current I (column 1, Table I). For our case, the values of k^2, thus obtained, were plotted against C_s/R (curve P), as shown in Fig. 3 where the computed values of C_s/R (curve LG) obtained from Liebmann and Grad's (1951) data are also shown for comparison.

CONCLUSION

It is shown that the experimental values of C_s/R, represented here, are consistently higher than the values of Liebmann *et al.* (1951), whose results are obtained analytically. However, at the maximum attainable value of the projector lens magnification 226*X*, the amount of distortion is 8.9 per cent, which seems to be a very reasonable value So, it may be concluded that, from the practical point

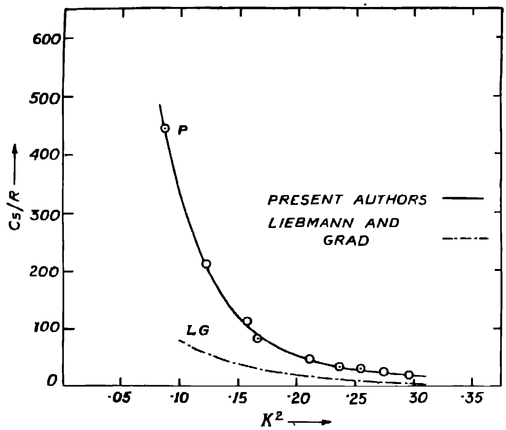

Fig. 1 Variation of C_s/R with the excitation parameter, k^2

of view, the values of spherical aberration constants at different magnifications, that are represented here, may be used with more confidence than those obtained analytically

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